

Scrap charge optimization using fuzzy chance constrained linear programming

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Abstract

We consider the problem of determining the optimal mix of different kinds the scrap in steel production. The uncertainty of the chemical composition of different kinds of scrap induces a considerable risk for the scrap mix failing to satisfy the composition requirements for the final product. We formulate the scrap charge optimization problem as a fuzzy chance constrained linear programming problem. We adopt a strengthened version of soft constraints to interpret the fuzzy constraints based on the application context and form a crisp model with consistent and compact constraints for solution. The simulation results based on the realistic steel production data show that the failure risk can be hedged by proper combination of aspiration levels and confidence factors for representing the fuzzy number. There is a tradeoff between failure risk and material cost. The presented approach applies also for other scrap-based production processes, such as aluminum and copper production.

Keywords: Fuzzy sets, linear programming, chance constraint, scrap charge optimization, steel production.

1 Introduction

A general trend during the past decades is that scrap-based steelmaking has increased its share, reaching around 40% of global crude steel production in 2001 (Rautaruukki 2001). Steel is also the world's most important recycled material. The use of the steel scrap as a raw material for steelmaking results in a saving of 600 million tonnes of iron ore and 200 million tonnes of coke each year (EUROFER 2001). With the growing concern on environmental issue, the popularity of using scrap could further increase because scrap-based steelmaking emits significantly less CO₂ as compared with integrated steelmaking using metallurgical coke as reductant for iron-making. Undoubtedly, the use of the scrap offers the opportunity to produce high quality products most economically. In the meantime, it also poses challenge in charge optimization caused by the uncertainty of chemical composition of scrap. The uncertainty mainly comes from two sources.

First, the constituents in the scrap and steel product are diverse. With the development of steel products for higher grades and higher performance, use of steel materials in combination with nonferrous metals or non-metallic materials has increased. Depending on the melting process and the requirements of the particular product, some of the constituents in scrap are considered impurities while others are valuable additives.

Second, the diverse scrap materials are generally divided into scrap types for handling and the materials included in each scrap type are heterogeneous because the classification varies based on different criteria. That means there are large deviations in material properties inside class, sometimes larger than between classes. Therefore, it is difficult to come up with accurate chemical composition (element concentration) analysis for the scrap.

Scrap-based steelmaking process starts with the charge of the predetermined scrap mix in an electric arc furnace (EAF). One furnace of refined steel (with required chemical composition) is called a *heat*. When the scrap mix is melted, there is a considerable risk for the outcome failing to satisfy the composition requirements for the product because of the uncertainty in the chemical composition for the scrap. The objective of the scrap charge optimization is to select the most economical scrap mix for each produced steel grade while minimizing the failure risk caused by uncertainties in raw material consistency.

There are three kinds of charge optimization models based on different planning horizon: single-heat model (AFS 1986) for short-term planning (e.g. daily operations), multi-heat model (Kim & Lewis 1987) for medium-term planning (e.g. weekly or monthly) and product recipe model (Lahdelma et al. 1999) for long-term planning (annually). Any charge plan is implemented on single-heat basis and on-line analysis is applied to identify whether the

outcome match the predicted characteristics and to correct the possible bias in future predictions (Wilson et al. 2001). However, if the charge planning is designed using the single-heat model, this may result in non-optimal use of raw materials. Minimizing raw material costs in one heat can eat up the cost-efficient raw materials and therefore increase the costs in future heats. The multi-heat model can allocate the available raw materials optimally among the different heats based on the current raw material stock and predicted deliveries. For long-term planning, it is more convenient to use product recipe model which can be viewed as a model where all heats of the same product are grouped. Therefore, the product recipe model is much smaller than the multi-heat model. The long-term scrap charge plan is designed based on forecast customer orders and possible available raw materials. The product recipe model can also be applicable to medium-term planning.

In terms of handling uncertainty, there are several methods of constraining failure risks. Bliss (1997) added safety margins to the product standard by using a tighter product standard in optimization. Lahdelma et al. (1999) added safety margins to the chemical composition for scrap in optimization. The approach of adding safety margin can be viewed as the extension of the deterministic model to accommodate the uncertainty. Turunen (2000) constrained the failure risk based on stochastic chance constraints to guarantee that the failure rate is less than a predetermined level (or to allow small violations in some constraints). However, stochastic chance constrained programming models are difficult to solve since the deterministic version of the model is non-linear (Kall & Wallace 1994, Watanabe & Ellis 1994). The solution can be even more complicated if the stochastic model includes simultaneously chance constraints and ordinary constraints with stochastic parameters.

In this paper, we formulate the scrap charge optimization problem based on the product recipe model. We represent the uncertainty for the scrap composition and the composition specification for product standard based on fuzzy set theory (possibility theory, Zadeh 1978). Then we constrain the failure risk based on a possibility measure. Consequently, the scrap charge optimization problem is modeled as a fuzzy linear chance constrained programming problem. This approach can be viewed as explicitly integrating methods of adding safety margin for product standard and for scrap composition and allowing small violations (chance constraints) in some constraints under a single framework. Since the constraints in the scrap charge problem mainly address the specification of the product, the crisp equivalence of the fuzzy constraints should be less relaxed than that purely based on the concept of soft constraints. For general discussions on the interpretation of the ordinary fuzzy constraints based on the concept of soft (flexible, approximate) constraints, we refer to Canz (1996), Dubois & Prade (1980), Slowinski (1986), Werners (1987), Zimmermann

(1978). For interpretation of general chance constraints directly based on the possibility measure, we refer to Liu & Iwamura (1998). Here we interpret chance constraints based on the likelihood measure and the resulting crisp constraints are much stricter than those based directly on the possibility measure. The property of the likelihood measure introduced in this paper is similar to that of the likelihood profile discussed in the fuzzy machine scheduling context (Wang et al. 2002). There are two ways to interpret ordinary fuzzy constraints: soft constraints and tolerance constraints. The strict tolerance constraint (Dubois & Prade 1980) means that the fuzziness of the right-hand side of the constraint is interpreted as a maximum tolerance for that of the left-hand side of the constraint. However, the system with the strict tolerance constraints may be inconsistent. Therefore, we interpret the ordinary fuzzy constraints based on the framework of relaxed tolerance constraints where we try to eliminate the possible conflicts of the strict tolerance constraints by dropping the relatively less critical constraints or by weakening some constraints. Finally a crisp model with consistent and compact constraints is formed. The resultant crisp model is a standard linear programming (LP) model, which can be solved by standard LP software.

The paper organizes as follows. In Section 2, we review a generic fuzzy linear programming model and describe the relationship between the fuzzy number and the stochastic parameter with given distribution. In Section 3, we describe the scrap-based steelmaking process first, and then we formulate the scrap charge optimization problem as a fuzzy chance-constrained linear programming model. In Section 4, we transform the fuzzy model into a crisp model with compact and consistent constraints based on the application context. In Section 5, we report the simulation results.

2 Fuzzy linear programming model and fuzzy numbers

We define fuzzy linear programming (FLP) as the extension of the classical linear programming (LP) in operations research (Taha 1992) in the presence of uncertainty in the optimization process where some or all of the coefficients are represented as fuzzy quantities (numbers) based on fuzzy set theory (possibility theory). A non-fuzzy quantity can be viewed as a special case of fuzzy quantity as discussed later. A generic FLP model is given below.

$$\min \sum_{j=1}^n \tilde{c}_j x_j \quad (1)$$

s.t.

$$\text{Pos} \left\{ \sum_{j=1}^n \tilde{a}_{ij} x_j \leq \tilde{b}_i \right\} \geq \alpha_i, \quad i = 1, \dots, m_1, \quad (2)$$

$$\sum_{j=1}^n \tilde{d}_{ij} x_j \leq \tilde{e}_i, \quad i = m_1 + 1, \dots, m, \quad (3)$$

$$x_j \geq 0, j=1, \dots, n, \quad (4)$$

where x_j are decision variables, \tilde{c}_j are cost coefficients, $\tilde{a}_{ij}, \tilde{d}_{ij}$ are technical coefficients, and \tilde{b}_i, \tilde{e}_i are right-hand side coefficients. Some or all of these coefficients can be fuzzy numbers. Formula (1) is the objective function to be optimized. Constraints (2) are called chance-constraints. $Pos\{.\}$ denotes the possibility of events $\{.\}$. The predetermined aspiration level α_i is usually interpreted as constraint reliability. The constraints (2) mean that the possibility for violating $\sum_{j=1}^n \tilde{a}_{ij} x_j \leq \tilde{b}_i$ should be less than $1-\alpha_i$. Constraints (3) are ordinary linear constraints.

Fuzzy numbers play a central role in the fuzzy programming model. First, we review the concept of fuzzy numbers briefly. Then we represent the statistical uncertainty based on fuzzy set theory and establish the relationship between the fuzzy number and the stochastic parameter with given distribution.

A fuzzy number is a convex normalized fuzzy set A on the real line R with membership function $f_A(x)$ ($x \in R$). We represent the fuzzy number based on the modified L - R fuzzy number. The examples of L - R fuzzy numbers can be referred to Dubois & Prade (1980) and Slowinski (1986). The representation for A is the 3-tuple of parameters $A=(a, \underline{a}, \bar{a})$, where $a, a-\underline{a}$ and $\bar{a}-a$ are mean, left and right spreads of the fuzzy number. The member function $f_A(x)$ is given below.

$$f_A(x) = \begin{cases} L((a-x)/(a-\underline{a})) & \text{if } \underline{a} \leq x \leq a, \\ R((x-a)/(\bar{a}-a)) & \text{if } a \leq x \leq \bar{a}, \\ 0 & \text{otherwise,} \end{cases} \quad (5)$$

where L and R are continuous decreasing functions such that $L(0) = R(0) = 1, L(1) = R(1) = 0$. If both L and R functions are linear, then the resultant fuzzy number is called triangular fuzzy number as shown in Figure 1, where f_A^L and f_A^R are L and R functions and $f_A^L(u) = f_A^R(u) = 1-u, 0 \leq u \leq 1$. A non-fuzzy number can be treated as a special case of the fuzzy number where both the left and right spreads are zero.

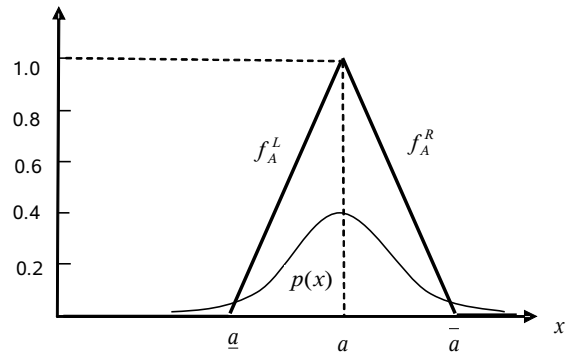


Figure 1 Relationship between a fuzzy number $A=(a, \underline{a}, \bar{a})$ and a stochastic parameter with density function $p(x)$ (a is the mean of the stochastic parameter).

Figure 1 also illustrates the relationship between a fuzzy number $A=(a, \underline{a}, \bar{a})$ and a stochastic parameter with density function $p(x)$. The uncertainty involved in many applications can be viewed as statistical uncertainty. Canz (1996) discussed the basic approach for representing statistical uncertainty using trapezoidal fuzzy numbers. Here, we follow the similar logic and represent the statistical uncertainty using modified L - R fuzzy numbers. First we assume a probability density function $p(x)$ from the statistical data, and then we transform it into a possibility distribution (fuzzy set) based on the shape of $p(x)$ and the mean and standard deviation of the statistical data. If the mean and standard deviation of the density function $p(x)$ is β and σ respectively, then for a fuzzy number $A=(a, \underline{a}, \bar{a})$, $a = \beta \pm \varepsilon$, $\underline{a} = a - n_1 \sigma$ and $\bar{a} = a + n_2 \sigma$, where ε is related to the skewness (the third moment, Banks & Carson 1984) of the probability density function $p(x)$, n_1 and n_2 are confidence factors. The choice of n_1 and n_2 depends on shape of $p(x)$ and the application context. If the density function $p(x)$ is skewed (e.g. log normal distribution), then n_1 and n_2 should be different. If $p(x)$ is symmetric (e.g. normal distribution), then $a = \beta, n_1 = n_2$. However, if the parameter has non-negativity restriction then $\underline{a} = \max(0, a - n_1 \sigma)$. If $a - n_1 \sigma < 0$, this is equivalent to case that n_1 and n_2 take different values. The value setting of n_1 and n_2 depends on the requirement of the application. The choice of L and R functions is flexible and simple linear functions can satisfy the needs for most applications.

3 Problem description

To understand the scrap charge optimization problem, we start by introducing the scrap-based steel-making process, e.g. stainless steel-making process.

3.1 Scrap-based steel-making process

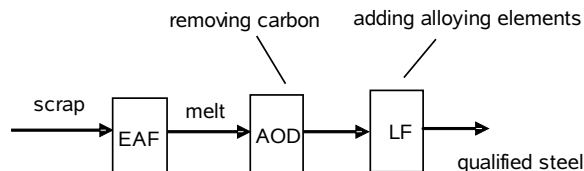


Figure 2. Scrap-based stainless steel-making process

Figure 2 shows a sketch of the scrap-based stainless steel-making process. The scrap metal is charged into an electric arc furnace (EAF) and melted. Then the melt is moved into argon-oxygen decarburization (AOD) converter where the carbon is removed by argon and oxygen injection in order to meet the steel quality requirement. After AOD, the molten steel is moved into ladle furnace (LF) for secondary metallurgical operations such as adding alloying elements, degassing and temperature homogenization.

The scrap consists of materials from different sources such as old cars, structural steel, cans, electric appliances and industrial by-products. Scraps contain also waste from the manufacturing process of steel products such as cut edges of steel plates and ends of coils. Generally the diverse materials are divided into several standard types for trade. After the scrap materials are delivered to the meltshop, the steel producers have their own internal classification system that further divides the standard types into different subtypes based on origin, supplier, chemical contents and size distribution and requirements. The total number of scrap types is about 20 for a normal carbon-steel meltshop. For stainless and speciality production, the classification can be even finer and the number of scrap types can be more than 100 (Sandberg 2005). For the stainless steel production, it is particularly important to sort grades with high content of valuable alloying metals such as nickel or molybdenum in their own classes so that the scrap can substitute for the expensive pure alloying materials to provide alloying elements in the new product. It would be good for the production if all different alloys could be separated to different classes but it seems that there are large deviations in properties (e.g. element concentrations) inside classes, sometimes larger than between classes. Some scrap classes are much more heterogeneous than others. That is, there is uncertainty for element concentration for the scrap. The uncertainty can cause errors for element concentrations in the final products.

The scrap also contains oxides, organic materials that burn in the process, and the metal itself forms oxides in the process. The *yield* of the raw material (material yield) is the share of the raw material that becomes part of the product, typically 70-95 %. We also use a yield parameter for each element (element yield coefficient). The element yield coefficients depend on the conditions in the furnace, while the material yield depends mainly on the physical and chemical properties of the material. The two yield parameters multiplied together give the net yield of

an element for a raw material. The yield and element concentrations of the scrap types can be estimated based on spectrographic analysis in conjunction with analysis of the process and heat data (Ikäheimo 1999, Sandberg 2005).

The process conditions may also vary and produce errors not related to scrap. Inaccuracies in the charging process constitute a third source of error. However, the element concentrations for the scrap are the most significant source of uncertainties in the process. We can treat this uncertainty as the sources for all the errors.

3.2 Scrap charge optimization problem

The objective of the charge optimization is to fulfill a set of customer orders at the lowest cost using available raw materials. The costs include the raw material cost and an approximation of the costs in successive process stages (the AOD converter and LF). Charge optimization models usually have constraints for the chemical balance of the steel, production amounts and material resources (Kim & Lewis 1987). Here we also consider the uncertainty in element concentrations for different scrap types. The uncertainty of the element concentrations for the scrap will cause the element concentration in the final product to deviate from the product standard. The product standard is specified by lower and upper limits for each alloying element concentration. A failure happens when the concentrations of the alloying elements of the molten steel in EAF exceeds the upper limits of the product standard because it is usually difficult to remove any element from the liquid steel. Falling below lower limits is not as critical because the element contents can be increased by adding pure alloying elements. However, it causes cost increase. On the one hand, the raw material cost will increase because the pure alloying materials are more expensive than the scrap. On the other hand, additional processing cost will be incurred in the subsequent stages in LF because the processing time will be increased. Therefore, the ideal scrap mix would yield the element concentrations that are close to the lower bounds of product standard.

The orders are divided into $|K|$ batches and each batch consists of products of the same standard. The element concentrations of raw materials and the element concentration requirements of final product are represented as modified *L-R* fuzzy numbers. We constrain the failure risk based on the possibility measure to control the failure rate within the predetermined level. The following notations are introduced.

Index Set

- I Set of all useful elements in the product including element iron.
- I_1 Set of alloying elements.
- J Set of all raw materials, $J = J_0 \cup J_1$.
- J_0 Set of scrap materials.

J_1 Set of pure alloying materials, each of which is made up of one alloying element.

K Set of batches.

Parameters

a_j Yield of raw material $j \in J$.

b_i Yield coefficient of element $i \in I$.

c_j Cost of raw material $j \in J$. For the alloying materials $j \in J_1$, additional processing cost is included in material cost.

m_k Mass of products in batch $k \in K$.

$\underline{x}_{k,j}, \bar{x}_{k,j}$ Lower and upper bounds for charge of raw material $j \in J$ in batch $k \in K$.

$\underline{x}_j, \bar{x}_j$ Lower and upper bounds for usage (consumption) of raw material $j \in J$.

$\lambda_{k,i}$ Aspiration levels for controlling the upper limit of the concentration of alloying element $i \in I_1$ in batch $k \in K$.

\tilde{p}_{ji} ($p_{ji}, \underline{p}_{ji}, \bar{p}_{ji}$) Concentration of element $i \in I$ in raw material $j \in J$ (fuzzy number).

\tilde{t}_{ki} ($t_{ki}, \underline{t}_{ki}, \bar{t}_{ki}$) Concentration of alloying element $i \in I_1$ in batch $k \in K$ (fuzzy number).

Decision variables

x_{kj} Charge (consumption) of material $j \in J$ in batch $k \in K$.

Then the scrap charge optimization problem for minimizing the raw material costs including additional processing costs is represented as follows.

$$\min \sum_{k \in K} \sum_{j \in J} c_j x_{kj} \quad (6)$$

s.t.

$$\sum_{j \in J_0} b_i a_j \tilde{p}_{ji} x_{kj} \leq m_k \tilde{t}_{ki}, \quad k \in K, i \in I_1, \quad (7)$$

$$Pos \left\{ \sum_{j \in J_0} b_i a_j \tilde{p}_{ji} x_{kj} \leq m_k \bar{t}_{ki} \right\} \geq \lambda_{ki}, \quad k \in K, i \in I_1, \quad (8)$$

$$\sum_{j \in J} b_i a_j \tilde{p}_{ji} x_{kj} = m_k \tilde{t}_{ki}, \quad k \in K, i \in I_1, \quad (9)$$

$$\sum_{i \in I} \sum_{j \in J} b_i a_j \tilde{p}_{ji} x_{k,j} = m_k, \quad k \in K, \quad (10)$$

$$\underline{x}_j \leq \sum_{k \in K} x_{kj} \leq \bar{x}_j, \quad j \in J, \quad (11)$$

$$\underline{x}_{kj} \leq x_{kj} \leq \bar{x}_{kj}, \quad k \in K, j \in J. \quad (12)$$

Constraints (7)–(9) together are used to control the concentrations of alloying elements for raw materials. Constraints (7) are general requirements for the concentrations of the alloying elements for the scrap materials. The concentrations of the alloying elements for the scrap in EAF cannot exceed the product standard. Constraints (8) are chance constraints particularly constraining the failure risk based on a possibility measure. The failure possibility that the concentrations of the alloying elements for the scrap exceed the upper limits of the product standard must be less than the predetermined levels $1 - \lambda_{ki}$ for each alloying element $i \in I_1$ in each

batch $k \in K$. The effect of the chance constraints is two-fold. On the one hand, they emphasize that controlling the upper limits of the alloying element concentrations is critical. On the other hand, they give flexibility to control the limits by choosing different aspiration levels, allowing explicitly small violations of the upper limit of the product standard. Constraints (9) state that the concentrations of the alloying elements in the final product must meet the product standard. Constraints (10) state the mass balances between raw materials and final products. Here the constraints are crisp. We should know that the summation of concentrations of all elements must be one in the original raw materials. Some elements may be burnt off in the process and the summation of concentrations of all elements that become part of product is fixed for a given material. The mass is the aggregate of the share of all elements that become part of products for the selected materials. Therefore, the uncertainty in element concentrations has only slight effect on mass because the increase in concentration for one element implies the decrease in concentration for other elements. The variations in mass mainly come from the possible different element yield coefficient b_i for different element $i \in I$. Constraints (11) and (12) give the bounds for raw material consumption.

4 Transformation of the fuzzy model into its crisp equivalence

In the fuzzy environment, there are two ways to treat the ordinary fuzzy constraints: tolerance constraints and soft (approximate) constraints (Dubois & Prade 1980). Tolerance constraints mean that the fuzziness of the right-hand side of the constraint is interpreted as a maximum tolerance for that of the left-hand side of the constraints. The soft constraints mean that the satisfaction of the constraints is determined by the membership function and there is compensation between the satisfaction of the constraints and fulfillment of the objective. That is, the interpretation of the fuzzy constraints based on the concept of soft constraints results in the crisp constraints that are more relaxed than the corresponding constraints assuming that there is no fuzziness in the coefficients, while the crisp constraints based on the concept of tolerance constraints are much stricter. Based on this logic, we can find later the interpretation of the chance constraints directly based on the possibility measure follows the line of soft constraints.

In case of multi-criteria decision environment where criteria are described by objective functions and possible alternatives are implicitly determined by constraints, it is reasonable that the fuzzy constraints are interpreted as soft constraints which can provide a certain degree of freedom in determining the set of feasible solutions. However, for the scrap charge problem, the interpretation of the fuzzy constraints should be stricter than that based on the concept of soft constraints because the constraints mainly impose physical restrictions on the product and must

be somewhat strictly satisfied. Therefore, we interpret the chance constraints based on the likelihood measure to obtain a stricter interpretation than that based directly on possibility measure. We interpret the ordinary fuzzy constraints based on the framework of the relaxed tolerance constraints where we try to eliminate the possible conflicts from the strict tolerance constraints by dropping the less critical constraints or by weakening some constraints. For simplicity, we assume that L - R fuzzy numbers are triangular. Different L and R function types mainly affect the setting of the aspiration level and complexity for the realization of chance constraints.

4.1 Interpretation of chance constraints

Based on the operation of fuzzy numbers

$$\sum_{j \in J_0} b_i a_j \tilde{p}_{ji} x_{kj} = \left(\sum_{j \in J_0} b_i a_j p_{ji} x_{kj}, \sum_{j \in J_0} b_i a_j \underline{p}_{ji} x_{kj}, \sum_{j \in J_0} b_i a_j \bar{p}_{ji} x_{kj} \right), \quad (13)$$

where \tilde{w}_{ik} is also a triangular fuzzy number.

Then constraints (8) become

$$Pos \{ \tilde{w}_{ki} \leq m_k \bar{t}_{ki} \} \geq \lambda_{ki} \quad (14)$$

Let $f_{\tilde{w}_{ki}}(x)$ be the membership function of the fuzzy number \tilde{w}_{ik} (Figure 3). $f_{\tilde{w}_{ki}}^L(u) = f_{\tilde{w}_{ki}}^U(u) = 1 - u$, $u \in [0, 1]$ are L and R functions. Then based on (5) $f_{\tilde{w}_{ki}}(x)$ can be represented as follows.

$$f_{\tilde{w}_{ki}}(x) = \begin{cases} f_{\tilde{w}_{ki}}^L((w_{ki} - x)/(w_{ki} - \underline{w}_{ki})) & \text{if } \underline{w}_{ki} \leq x \leq w_{ki}, \\ f_{\tilde{w}_{ki}}^U((x - w_{ki})/(\bar{w}_{ki} - w_{ki})) & \text{if } w_{ki} \leq x \leq \bar{w}_{ki}, \\ 0 & \text{otherwise,} \end{cases} \quad (15)$$

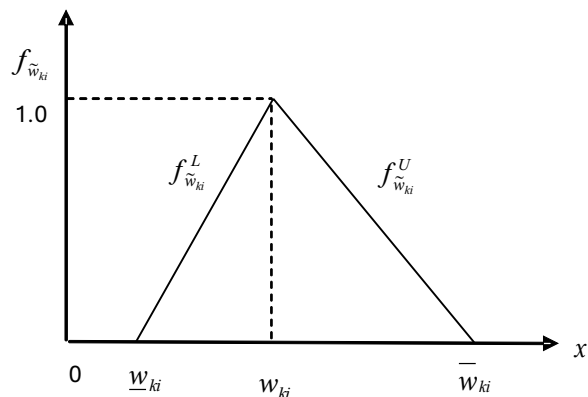


Figure 3. The membership function of a fuzzy element concentration \tilde{w}_{ki} .

Then the left hand side of (14) can be calculated as follows.

$$Pos \{ \tilde{w}_{ki} \leq m_k \bar{t}_{ki} \} = \sup \{ f_{\tilde{w}_{ki}}(x) \mid x \leq m_k \bar{t}_{ki} \} = \begin{cases} 0 & \text{if } x \leq \underline{w}_{ki}, \\ f_{\tilde{w}_{ki}}^L((w_{ki} - x)/(w_{ki} - \underline{w}_{ki})) & \text{if } \underline{w}_{ki} \leq x \leq w_{ki}, \\ 1 & \text{if } x \geq \bar{w}_{ki}. \end{cases} \quad (16)$$

If the realization of (14) is directly based on (16), we can see that any aspiration level λ_{ki} can be satisfied by the value (element concentration) less than w_{ki} . This implies the concept of soft constraints. However, this realization is too relaxed for the scrap charge problem because it cannot control failure rate effectively.

To enforce the chance constraints more strictly, we construct the likelihood of the *valid* element concentration with fuzzy possibility distribution. We define the likelihood of the valid element concentration for a fuzzy element concentration in the similar way as Wang et al. (2002) defined the job completion likelihood profile for the fuzzy processing time in the machine scheduling context.

Let $\mu_{\tilde{w}_{ki}}$ denote the likelihood of the *valid* element concentration for a fuzzy element concentration \tilde{w}_{ki} . For a given \tilde{w}_{ki} , $\mu_{\tilde{w}_{ki}}$ represents the likelihood of its element concentration to be valid within a certain allocated upper limit y (variable) for the product standard. If $\underline{w}_{ki} \geq y$, the element concentration is invalid and $\mu_{\tilde{w}_{ki}} = 0$. If $\bar{w}_{ki} \leq y$, then the element concentration is completely valid $\mu_{\tilde{w}_{ki}} = 1$. When $\underline{w}_{ki} \leq y \leq \bar{w}_{ki}$, $\mu_{\tilde{w}_{ki}}$ should vary

from 0 to 1 based on an increasing function. We wish that $\mu_{\tilde{w}_{ki}} : [\underline{w}_{ki}, \overline{w}_{ki}] \leftrightarrow [0, 1]$ is a continuous bijective mapping, i.e. there is only element concentration in $[\underline{w}_{ki}, \overline{w}_{ki}]$ corresponding to a given aspiration level $\mu_{\tilde{w}_{ki}} = \lambda_{ki}$. $\mu_{\tilde{w}_{ki}}$ can be constructed based on $f_{\tilde{w}_{ki}}$ as follows (Figure 4).

$$\mu_{\tilde{w}_{ki}}(y) = \begin{cases} 0 & \text{if } y \leq \underline{w}_{ki}, \\ f_{\tilde{w}_{ki}}^L((w_{ki} - y)/(w_{ki} - \underline{w}_{ki}))/2 & \text{if } \underline{w}_{ki} \leq y \leq w_{ki}, \\ 2 - f_{\tilde{w}_{ki}}^U((y - w_{ki})/(\overline{w}_{ki} - w_{ki}))/2 & \text{if } w_{ki} \leq y \leq \overline{w}_{ki}, \\ 1 & \text{if } y \geq \overline{w}_{ki} \end{cases}$$

$$\mu_{\tilde{w}_{ki}}(y) = \begin{cases} 0 & \text{if } y \leq \underline{w}_{ki}, \\ f_{\tilde{w}_{ki}}^L((w_{ki} - y)/(w_{ki} - \underline{w}_{ki}))/2 & \text{if } \underline{w}_{ki} \leq y \leq w_{ki}, \\ 2 - f_{\tilde{w}_{ki}}^U((y - w_{ki})/(\overline{w}_{ki} - w_{ki}))/2 & \text{if } w_{ki} \leq y \leq \overline{w}_{ki}, \\ 1 & \text{if } y \geq \overline{w}_{ki} \end{cases} \quad (17)$$

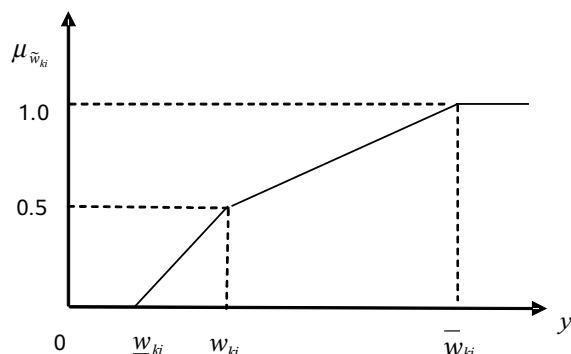


Figure 4. The likelihood of the valid element concentration for a fuzzy element concentration \tilde{w}_{ki} .

Our application background requires that $\lambda_{ki} > 0.5$ for the chance constraints. This means that

$$\lambda_{ki} = (2 - f_{\tilde{w}_{ki}}^U((y - w_{ki})/(\overline{w}_{ki} - w_{ki}))/2) / f_{\tilde{w}_{ki}}^U((y - w_{ki})/(\overline{w}_{ki} - w_{ki})) = 1 - (y - w_{ki})/(\overline{w}_{ki} - w_{ki}) \Rightarrow y = w_{ki} + (2\lambda_{ki} - 1)(\overline{w}_{ki} - w_{ki}) \quad (18)$$

Then the crisp equivalence of chance constraints (8) based on (13) and (18) is given below.

$$\sum_{j \in J_0} b_i a_j (p_{ji} + (2\lambda_{ki} - 1)(\overline{p}_{ji} - p_{ji})) x_{kj} \leq m_i \quad k \in K, i \in I_1. \quad (19)$$

4.2 Interpretation of ordinary constraints

For a generic ordinary inequality fuzzy constraint

$$\sum_j \tilde{d}_{ij} x_j \leq \tilde{e}_i, \quad (20)$$

where \tilde{d}_{ij} and \tilde{e}_i are L - R fuzzy numbers, $\tilde{d}_{ij} = (d_{ij}, \underline{d}_{ij}, \overline{d}_{ij})$ and $\tilde{e}_i = (e_i, \underline{e}_i, \overline{e}_i)$. If a fuzzy constraint is interpreted as a tolerance constraint, then crisp equivalence is given below (Dubois & Prade 1980).

$$\begin{aligned} \sum_j d_{ij} x_j &\leq e_i, \\ \sum_j (\overline{d}_{ij} - d_{ij}) x_j &\leq \overline{e}_i - e_i, \\ \sum_j (d_{ij} - \underline{d}_{ij}) x_j &\leq e_i - \underline{e}_i, \end{aligned} \quad (21)$$

where the first, second, and third constraints impose the constraints in terms of mean, right and left spread respectively. This means that one fuzzy constraint in principle should be transformed into three crisp constraints.

To enforce the right spread constraint in (21), we can combine the first and second constraints in (21) and obtain

$$\sum_j \overline{d}_{ij} x_j \leq \overline{e}_i \quad (22)$$

To enforce the left spread constraint in (21), we can combine the first and third constraints in (21) and obtain

$$\sum_j \underline{d}_{ij} x_j \geq \underline{e}_i \beta_i, \quad 0 < \beta_i \leq 1, \quad (23)$$

where β_i is a scaling factor.

However, simultaneously enforcing the three constraints in (21) may cause inconsistency; especially the third constraint may conflict with the first two because the third constraint imposes the restriction in the opposite direction of the first two (22). For the current charge problem, we can choose not to enforce the spread constraints, which we will discuss a little bit later.

If the constraint (20) is an equality constraint, it can be weakened to $\sum_j \tilde{d}_{ij} x_i \subseteq \tilde{e}_i$ in the sense of Zadeh (Dubois & Prade 1980). The crisp equivalence is similar to (21) with mean constraint (the first constraint) enforced as an equality constraint and the other two constraints remaining unchanged. Now we discuss the transformations of constraints (7) and (9) for the scrap charge problem.

In terms of constraints (7), when the right spread constraint (22) applies, it is in fact the chance constraint (19) with the aspiration level $\lambda_{ki}=1$. This is the strictest case of the constraint (19), implying that enforcing right spread constraints make the chance constraint lose the flexibility to control the upper limit of the element concentration based on different aspiration levels. Therefore, we choose not to enforce the right spread constraint. The left spread constraint for constraints (7) is used to enforce the lower limit of the element concentration. However, the lower limits are not critical. Moreover, enforcing the left spread constraints can only result in shrinking the feasible region and thus increasing the cost of raw materials. Therefore, the left spread constraints are not enforced either. Then constraints (7) can be transformed based on the mean constraints only.

$$\sum_{j \in J_0} b_i a_j p_{ji} x_{kj} \leq m_k t_{ki}, \quad k \in K, \quad i \in I_1. \quad (24)$$

In terms of equality constraints (9), it is unnecessary to enforce the spread constraints because adding pure alloying elements does not introduce new uncertainty. That is, we only need to enforce the mean constraints $\sum_{j \in J} b_i a_j p_{ji} x_{kj} = m_k t_{ki}$. For the batch k , the product is up to the standard as long as the concentration for element i falls between the lower limit t_{ki} and upper limit \bar{t}_{ki} . Therefore, we can weaken the mean constraints to

$$\sum_{j \in J} b_i a_j p_{ji} x_{kj} \geq m_k t_{ki}, \quad k \in K, \quad i \in I_1. \quad (25)$$

In practice, it is unnecessary to worry about the upper limit $m_k \bar{t}_{ki}$ because the high cost of pure alloying materials forces the consumption of the pure alloying materials to be minimized.

Finally, the crisp equivalence of the original fuzzy programming model (6)-(12) becomes the objective function (6) in conjunction with constraints (19), (24), (25) and (10)-(12). We can see that the number of the constraints in crisp model does not increase as compared with that in the original fuzzy model because we drop the unnecessary constraints for interpretation of strict tolerance based on the application context. The crisp equivalence bears some similarity with the model resulting from adding the safety margins for element concentrations for the raw materials (Lahdelma et al. 1999). For example, the interpretation of the chance constraints (19) is similar to the effect of adding safety margins for element concentrations for raw materials. However, differences exist between these two approaches. Adding safety margins is an approach to extending the deterministic model to accommodate uncertainty and the complicated penalty cost structure to punish the possible deviation of an intermediate target in EAF is kept in the extended model. The fuzzy model directly addresses the uncertainty and each constraint in the crisp equivalence has a clear link with the fuzzy constraint. Consequently, the crisp equivalent

of the fuzzy model is more compact than the extended deterministic model based on adding safety margins because no intermediate target and penalty cost is introduced in the interpretation of fuzzy constraints. That is, fuzzy approach is more straightforward. The crisp equivalence of the fuzzy model is a standard LP model which can be solved by the standard LP solver.

5 Numerical experiments

We have tested our model with modified process data from a Finnish steel plant for stainless steel production. The aim of the experiments is to reproduce the products based on the crisp equivalent model using the available materials and evaluate the uncertainty of element concentration on failure risk (failure rate) and material cost. The stainless products contain five main elements: Iron (Fe), Manganese (Mn), Chromium (Cr), Nickel (Ni), and Molybdenum (Mo). The candidate raw materials contain at least one of above five elements. To guarantee the existence of a feasible solution, we assume that the supply of pure materials that consist of only one of above five elements is unlimited. The concentrations of alloying elements for raw materials are represented by the mean and standard deviation. The concentrations of alloying elements for final products are specified by lower and upper limits. Table 1 gives the concentrations of alloying elements for some raw materials and Table 2 gives the concentrations of alloying elements for some final products.

Table 1 Concentrations of alloying elements for some raw materials

Material	Element concentrations (%)			
	Mn	Cr	Ni	Mo
FeCr-1	0.6±0	41±0	4.1±0	0±0
FeNi	0±0	5±0	30±0	0±0
Ferrite -scrap	1±0.27	12±1.9	1±0.14	0±0
LC-scrap	0±0	2±0.34	3±0.4	0±0
Acid-proof scrap	2±0.54	16±2.57	11±1.43	2±0.07
Scandust scrap	2±0.54	22±3.67	6±0.77	1±0.04

Table 2 Concentrations of alloying elements for some final products

Product	Element concentrations (%)			
	Mn	Cr	Ni	Mo
710-2	[1,1.4]	[16.5,16.9]	[6.35,6.55]	[0.65,0.8]
720-1	[1.6,1.8]	[18,18.4]	[8.5,8.65]	[0,0.4]
720-4	[1.4,1.6]	[18,18.4]	[10,10.2]	[0,0.4]
731-1	[1.6,1.8]	[17,17.4]	[9,9.15]	[0,0.4]
750-1	[1.4,1.6]	[16.7,17.1]	[11,11.15]	[2,2.2]
757-2	[1.5,1.8]	[16.6,17]	[10.5,10.7]	[2.5,2.7]

Next, we investigate failure risk and material cost based on stochastic simulation. The specific procedures are given below.

First, the fuzzy parameters such as the element concentrations for products and raw materials are realized. The realization of the concentration of element i for product k , $\tilde{t}_{ki} = (t_{ki}, \underline{t}_{ki}, \bar{t}_{ki})$, is straightforward. \underline{t}_{ki} , and \bar{t}_{ki} correspond to the lower and upper limits for element concentration specified in product standard respectively, and $t_{ki} = (\underline{t}_{ki} + \bar{t}_{ki})/2$. For the concentration of element i for raw material j , $\tilde{p}_{ji} = (p_{ji}, \underline{p}_{ji}, \bar{p}_{ji})$, we assume that element concentrations follow normal distribution and choose suitable confidence factors n_1 and n_2 . Then \tilde{p}_{ji} can be realized based on the mean and standard deviation of element concentrations for raw materials as discussed in Section 2.

Second, for a fixed aspiration level $\lambda_{ki} = \lambda$ for element i in product k , we solve the crisp equivalence of the fuzzy model presented in Section 4 and obtain the selection of raw materials and related amount for producing a set of products.

Finally, stochastic simulations are performed. Random element concentrations for scrap are generated based on normal distribution. Then based on the material selection and related amount in the second step, the contribution of the scrap to the alloying elements in the product is computed. If the scrap contribution has already exceeded the upper limit of the product standard, then the failure is recorded. Otherwise, then amount of pure alloying materials is determined and the cost of raw materials is computed.

We reproduce 11 products using 17 raw materials (13 scraps + 4 pure alloying materials). We test two scenarios with different uncertainty degrees. The uncertainty degree is represented by the ratio of mean and standard deviation of the element concentrations. In the first scenario (S1), the maximum uncertainty degree among all the element concentrations in all materials is moderate and about 30%. In the second scenario (S2), the uncertainty degree for all the element concentration in all materials is doubled as compared with that of the first one. That is, the mean of each element concentration is same as that of the first scenario while the standard deviation of each element concentration is doubled as compared with that of the first scenario. We choose 6 aspiration levels and set confidence factors to 3. For each aspiration level in each scenario, ten turns of simulations are run. In each turn, 10^6 sets of random element concentrations are generated for the selected materials and then the failure rate and material cost is computed based on 10^6 outcomes. Then we aggregate the results from ten turns of simulation to obtain average failure rate and material costs. Table 3 gives the average failure rate (FR) and normalized cost (NC) for all of raw materials. The normalized cost uses the cost of the total raw materials for aspiration level $\lambda=1$ in the first scenario as reference.

Table 3 Average failure rate (FR, percentage) and normalized cost (NC, percentage) for two scenarios at different aspiration levels.

λ	S1		S2	
	FR	NC	FR	NC
0.75	21.96	98.49	24.45	100.61
0.8	11.81	98.63	10.53	101.08
0.85	5.41	98.84	4.71	101.52
0.9	2.20	99.17	1.80	101.92
0.95	0.76	99.58	0.61	102.29
1	0.10	100	0.25	102.63

Based on Table 3, the failure rate can be controlled by the proper combination of confidence factors (n_2) and aspiration levels (λ) regardless of the uncertainty degree. Here we can induce that the failure rate is unacceptable when $\lambda < 0.5$ because it is too large. Therefore, the interpretation of chance constraints directly based on the possibility measure (16) is not sufficient in this context. Based on formula (19) and the representation of fuzzy number, $(2\lambda-1)n_2$ can be interpreted as the confidence factors for the aggregated element concentrations for raw materials. The choice of n_2 is associated with the uncertainty degree for element concentration and the selection of raw materials. For scenario S1, the minimum failure rate is 0.25% for $n_2 = 3$. If we want to decrease the failure rate further, we should increase n_2 for representing the fuzzy number. For the same $(2\lambda-1)n_2$, the failure rate is of the same order regardless of the uncertainty degree. That means, $(2\lambda-1)n_2$ plays a central role in controlling failure rate.

In terms of material cost, on the average we can see that costs increase by about 1.15% from failure rate about 5% (aspiration level 0.85) to less than 0.3% (aspiration level 1) for both scenarios. When the uncertainty degree doubles (from S1 to S2), the raw material costs increase by about 2.5% for the same order of failure rate. That is, there is a tradeoff between the decrease of the failure rate and increase of material cost. When the uncertainty degree increases, the material cost need to increase to realize the same order of failure rate. When the failure rate needs to decrease for the same scenario, the material cost also needs to increase. We know that the material cost accounts for the major cost for steel production based on the recycled scrap. If the material cost involves a large monetary value, even 0.5% cost variation is significant. The loss incurred by failure is up to the production process and the degree of violations. For slight violation, the failure may be recovered easily with moderate cost. For severe failure, the current heat becomes a waste and the production needs to be restarted. The decision about the tradeoff between material cost and failure rate is up to the decision maker and the production process.

Table 4 Cost share (CS) of pure alloying materials and the number of different types of scraps used for different scenarios at different aspiration levels.

λ	S1		S2	
	CS	Scraps	CS	Scraps
0.75	28.69	9	30.97	5

0.8	29.02	9	31.59	5
0.85	29.40	8	32.12	5
0.9	29.68	6	32.57	5
0.95	30.02	5	32.96	5
1	30.55	5	33.30	5

Finally we discuss the mechanism of controlling the failure rate. The model reallocates the materials based on the aspiration level. The reallocation includes the increase/decrease of scrap types and adjustment of the amount used for each material. Table 4 shows cost share (CS) of pure alloying materials and the number of different types of scraps selected for different scenarios at different aspiration levels. On average we can see that cost share for the pure alloying materials increases by about 1.15% from failure rate about 5% (aspiration level 0.85) to less than 0.3% (aspiration level 1) for both scenarios. When the uncertainty degree doubles (from S1 to S2) the cost share increases by about 2.5% for the same order of failure rate. Here the cost share variation for the pure alloying materials is coincident with the overall material cost variation shown in Table 3. This implies that the material cost increase for reducing the failure rate mainly attributes to the share increase of the pure alloying materials. Based on the number of scrap types selected for products, we can see the material selection can be widened as the uncertainty degree decreases.

The mechanism to control the failure rate is to increase the share of pure alloying materials in a cost-efficient way. For the same scenario, when the aspiration level decreases, the decrease in cost share of alloying materials means the increase in share of the scrap. This in turn means the overall uncertainty for the selected material increases. This results in simultaneous increase of the failure rate and decrease of the material cost because scrap is much cheaper than pure alloying material. For different scenarios, as the uncertainty degree increases, the share of the pure alloying materials must increase to realize the same the order of failure rate.

Above simulation results can provide several guidelines for the practical production. First, for the rough estimates of the element concentrations, a little bit larger uncertainty degree estimation can be used to produce the initial selection of scrap mix with less failure risk. Second, the scrap mix can be reselected with less material cost based on more precise estimation for element concentration as more process data are available. Finally, the scrap selection based on more reliable concentration estimation from large quantities of process data can be used to guide the material purchase.

6 Conclusions

Handling the scrap is one of the most important functions for scrap-based steelmaking. Using the wrong grades of scrap will result in higher raw material costs and difficulty in meeting product standard. The objective of the scrap charge optimization

is to select the most economical scrap mix for each produced steel grade while minimizing the failure risk caused by uncertainties in raw material consistence. In this paper, we first presented the fuzzy chance constrained model for the scrap charge optimization based on product recipe and then transformed it into a crisp model based on the application context for solution. Simulation shows that the failure risk can be hedged by the proper combination of the aspiration levels and confidence factors for representing the fuzzy number. The mechanism of controlling the failure risk against uncertainty is to increase the share of the expensive pure alloying materials in the controlled manner. Therefore, there is a tradeoff between controlling failure risk and increase of the material cost. The model can be used for both determining the scrap mix for short-term operation of the melting shops based on available materials and for guiding the material purchase for long-term planning. The model can also be applied in other scrap-based production processes such as aluminum and copper (Lahdelma 1998).

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